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Frequency Response of Standard Resistors

Basic Equivalent Circuit

The basic lumped-parameter equivalent circuit for a two-terminal resistor is that shown here. For this circuit:

$$Z = \frac{R + j\omega[L(1-\omega^2 LC) - R^2 C]}{(1-\omega^2 LC)^2 + (\omega RC)^2} = Rs + jXs \quad [1]$$

$$Y = \frac{R + j\omega[R^2 C - L(1-\omega^2 LC)]}{R^2 + (\omega L)^2} = Gp + Bp \quad [2]$$

From these we can get:

$$Rs = \frac{R}{(1-\omega^2 LC)^2 + (\omega RC)^2} \quad \text{and} \quad Rp = R[1 + (\omega L/R)^2] \quad [3]\&[4]$$

and also $$Q = \omega[(L/R)(1 - \omega^2 LC) - RC] \quad [5]$$

If we neglect the $\omega^2 LC$ terms, we can see that (for this simple circuit)
Rs is independent of L and decreases because of C, and
Rp is independent of C and increases because of L,
and Q is positive (inductive) if R small and negative (capacitive) if R is large.

The $\omega^2 LC$ terms are the beginnings of parallel resonance between L and lumped shunt C and usually very small even at 1 MHz except for wire-wound resistors, spool-wound types being particularly bad. However, these terms do cause the series resistance of low-valued 1442 resistors to increase at much higher frequencies. Note that to cause an *increase* in Rs, L must be greater than $R^2 C/2$.

Corrected Measurements (Open and Short Correction)

Measurement results depend also on the OPEN and SHORT used for the calibration of the fixture used. If we let $Zsc = j\omega Ls =$ and $Yoc = j\omega Co$ then the formulas for corrected measurements on the simple equivalent circuit are:

$$Rs = \frac{R\{1 + \omega^2 LsCo)}{[1-\omega^2 L(C-Co)]^2 + [\omega R(C-Co)]^2} \quad [6]$$

or

$$Rs \cong R\{1 + \omega^2[2L(C-Co) - R^2(C-Co)^2]\} \quad [6a]$$

$$Rp = \frac{R[(1+\omega^2 LsC)^2 + \omega^2[(L-Ls + \omega^2 LsLC)/R]^2}{\{1 + \omega^2 LsCo)} \quad [7]$$

or

$$Rp \cong R\{1 + \omega^2(L-Ls)^2/R^2]\} \quad [7a]$$

and

$$Q = \omega\{(L-Ls+\omega^2 LLsC)[1-\omega^2 L(C-Co)]/R - R(C-Co)(1-\omega^2 LsC)\} \quad [8]$$

or

$$Q \cong \omega[(L-Ls)/R - R(C-Co)] - \omega^3 L(L-Ls)(C-Co)/R \quad [8a]$$

where the simplified formulas assume $\omega^2 LsC$ is very small.

For the Cal Kit resistors C-Co and L-Ls are probably small because the OPEN and SHORT use the same geometry as the standards. For the GR900 units Ls is zero so that L-Ls is the 9 nH of the equivalent circuit, and C-Cs is the 3.4 pF of the standard minus the .172 pF (±.008 pF) of the WO open or about 3.23 pF.

Dielectric Loss in the Lumped Shunt Capacitance
If the lumped parallel capacitance is lossy, the resistance is effectively paralleled by a shunt conductance. If we assume the simple model shown here with C having a finite D value then replacing C with C(1-jD) we get the conductance DωC or

$$Rp = R/(1 + \omega DCR) \quad [9]$$

$$Q = -\omega RC/(1 + \omega DCR) \quad [10]$$

This could greatly decrease Rp. For example if the lossy capacitance across the resistor itself (in the epoxy) is 0.9pF then a D of .03 gives a decrease of 0.1% in the 5.9k resistor at 1MHz. This probably explains the large changes in the higher-value "DW" resistors that used epoxy. Rs is similarly affected but if Rs is used only for 50 ohms or less, the effect is small. The standard 1442 resistors probably use a low-loss radio cement or polystyrene cement. Note that the epoxy may have a higher dielectric constant making the value of C larger for the "DW" units.
Note also that D is not constant, but varies with frequency. However, it generally varies slowly with frequency so that it causes an error in Rp that does not have frequency raised to an even power as do the errors from simple lumped models. There can be effects even at very low frequencies.

Dielectric loss can be modeled by and infinite R-C network and any such R-C network will show a reduction in its effective parallel capacitance with frequency. If D is constant the change in capacitance between two frequencies is given by

$$\Delta C/C = -D(2/\pi)\ln(f2/f1)$$

Note that D is not necessarily constant and that this is a second-order effect.

Capacitance to Guard (Ground)
The GR900 resistors are two-terminal, but the Cal Kit units are guarded and there is some capacitance from the body of the resistors to ground which varies between units depending how close the resistor is to the guard track. If Cg is the effective capacitance from the center of the resistor to guard, see figure, then the resistor has an effective inductance $CR^2/4$ giving

$$Rp \cong R[1 + (\omega RCg)^2/16] \quad [11]$$

$$Q \cong \omega R[Cg/4 - C - C(\omega RCg)^2/16] \quad [12]$$

This is an small effect but may not be negligible effect in the highest-value Cal Kit resistor. It can easily be seen in long tubular resistors if they are near ground. Evenly distributed capacitance to ground gives higher order terms but they decrease rapidly if ωRCg is small.

Dielectric Loss in the Capacitance to Guard

The Cal Kit resistors could have loss in the capacitance to ground, Cg, which was considered above. This effectively causes a further increase in Rp. Here the loss is represented by Dg.

$$Rp \cong R[1 + \omega DgRCg/4 + (\omega RCg)^2(1+Dg^2)/16] \quad [13]$$

Compare this to equation [11] which has no loss. These added terms with D or D^2 factors would further increase Rp, particularly the first. The loss would be in the dielectric material used for the case of the Vishay resistor.

Distributed Capacitance Along Resistor

The GR 900 resistors also have distributed capacitance to the outer conductor which, in this case is not guard, but is one of the two terminals. If we again lump this as a capacitor from the center of the resistor we get this equivalent circuit. Again because this effect affects only high-valued resistors we neglect inductance.

$$Rp \cong R[1 - (\omega RCy)^2/16] \quad [14]$$

$$Q \cong -\omega R(Cy/4 + C) \quad [15]$$

This is the classic cause for the decrease in Rp of a resistor, but for some the dielectric loss (equation 9) is probably a much bigger effect. Evenly distributed capacitance gives higher order terms. Capacitance between points along the resistor has a similar effect.

Note that there is loss in this distributed capacitance which also would reduce Rp slightly. However, this capacitance, Cy, is usually much smaller than the total lumped capacitance, C, and there is a factor of 1/4 in the error term so that the effect of loss in Cy is relatively small compared to that of loss in C (equation 9).

Skin Effect

Skin Effect causes an increase in resistance and inductance in thick, low-resistance conductors at high frequencies. This would affect the resistance and inductance of the coaxial conductors of the GR1442 resistors at very high frequencies and perhaps affect the wire leads of the Cal Kit resistors but the R and L of these parts is very small compared to the total R and L. The resistors themselves would probably not be affected.

Combined Formulas

If we combine the above effects but limit ourselves to ω and ω^2 terms for R and up to ω^3 terms for Q we have:

$$Rs \cong R\{1 + \omega[CgCgR/4 - DCR] + \omega^2[2L(C-Co) - R^2(C-Co)^2 - LsCo - (DCR)^2 - (RCy)^2/4]\} \quad [16]$$

$$Rp \cong R\{1 + \omega[DgCgR/4 - DCR] + \omega^2[(L-Ls)^2/R^2 + Ls(2C-Co) - (RCy)^2/16 + (RCg)^2(1+Dg^2)/16]\} \quad [17]$$

$$Q \cong \omega\left[(L-Ls)/R - R(C-Co) + RCg/4 - RCy/4\right] - \omega^2 D(RC)^2 - \omega^3 L^2 C/R \qquad [18]$$

Here some small ω^3 terms for Q have been omitted. Also we have included the effects of D, Dg and Cy on Rs as well as on Rp even though they are probably not important at very low values resistance values where Rs is used. (Note that NIST uses Rs if Q is positive and Rp if Q is negative. For the 1442 resistors Q = 0 somewhere near 50 ohms.)

Calculating Parasitic Parameters from Measured Values

If the simple equivalent circuit we started with was truly valid, and if the L and C values were the same for all resistors of a given type, independent of resistance value, then it would be easy to calculate L and C from equation [5] using only the $\omega L/R$ term for low values and the ωRC for high values. However, we have seen above that the equivalent circuit is not that simple and we know that L and C are not completely independent of R value. The most difficult effects to handle are those due to dielectric loss which affect higher resistance values.

If we plotted Q and changes in R vs frequency on Log-Log paper, we ought to be able to get an idea of the size of coefficients of ω in both quantities. However, there are many error terms of the same power as we can see from the combined formulas above. To separate them requires some testing with numbers to see what terms could be important and what could not. It might help to make measurements at much higher frequencies in order to detect the higher-powered terms more easily. We have noted that the important terms for the GR900 resistors are not necessarily the same as those for the Cal Kit resistors.

Other Considerations

Calibration of the frequency errors of the Cal Kit resistors from values of the 1442s assumes that the measuring instrument is unaffected by the change in the test fixture used. There are two (known) possible sources of error.

First, the capacitance to ground from the IL and PL terminals must be very constant because a change in this capacitance will cause a change in the error caused by the limited gain of the inverting opamp at high frequencies (that is, its increasing input impedance). To keep this change small, **connect the GR900 outer terminal to the IH and PH bridge terminals.** Also, **new OPEN and SHORT zeros must be made for each fixture configuration**

Second, the GR900 fixture adds a small series inductance and resistance and a shunt capacitance that are common to all measurements using that fixture including the OPEN and SHORT. These have a small effect on the corrected measurements: both Rs and Rp are multiplied by

$$1 - \omega^2(LaCo - LsCa) + j\omega(RaCo - RsCa) \qquad [19]$$

where La, Ra and Ca are these small parameters, Rs and Ls the parameters of the SHORT and Co the capacitance of the open. If we assume that all inductances are less than 10 nH, all resistances less that 1 ohm and all capacitances less than 10 pF, these errors are negligible at 1 MHz (< 10 ppm).

Actually there is a larger inductance caused by mutual inductance between the cables and in the 1689 fixture and a larger shunt capacitance in the fixture and on the main board. These add to La and Ca in the above formula. However these will be removed by calibration if the *changes* in Co and Ls between the calibration OPEN and SHORT and the OPEN and SHORT used in measurements are small.

Finally, the cables cause attenuation of both Ex and Ix that varies with frequency. However, these are common to all measurements and affect all measurements equally and thus are removed by calibration if the calibration is made with the same cables that are used for measurements. **Thus a new calibration should be made if the cable length is changed** (at least at the higher frequencies).